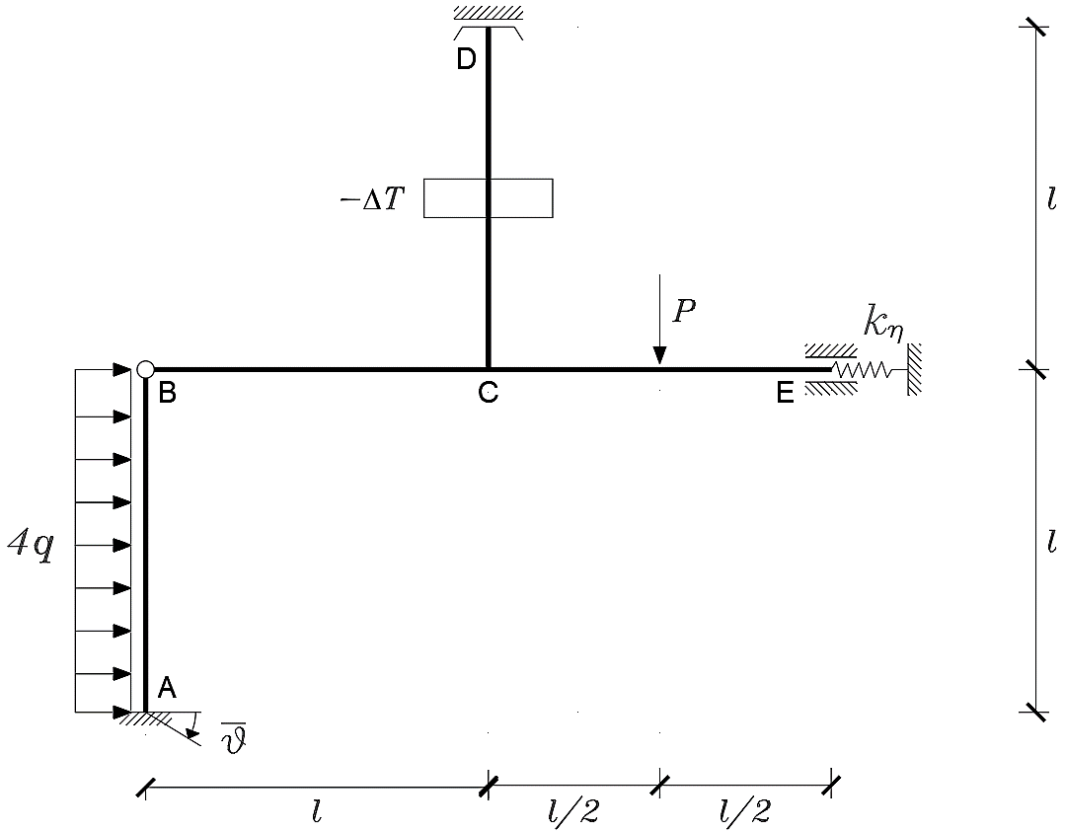
TECNICA DELLE COSTRUZIONI
TEMA ESAME DEL 22 NOVEMBRE 2021

DOCENTI:
PROF. GIOVANNI A. PLIZZARI
PROF. FAUSTO MINELLI

DURATA: **2 ORE E 30 MIN.**

Esercizio



$$k_\eta = 3\frac{EJ}{l^3} \qquad \bar{\theta} = \frac{ql^3}{EJ} \qquad \alpha\Delta T = \frac{1}{2}\frac{ql^3}{EJ} \qquad P = 4\,ql$$

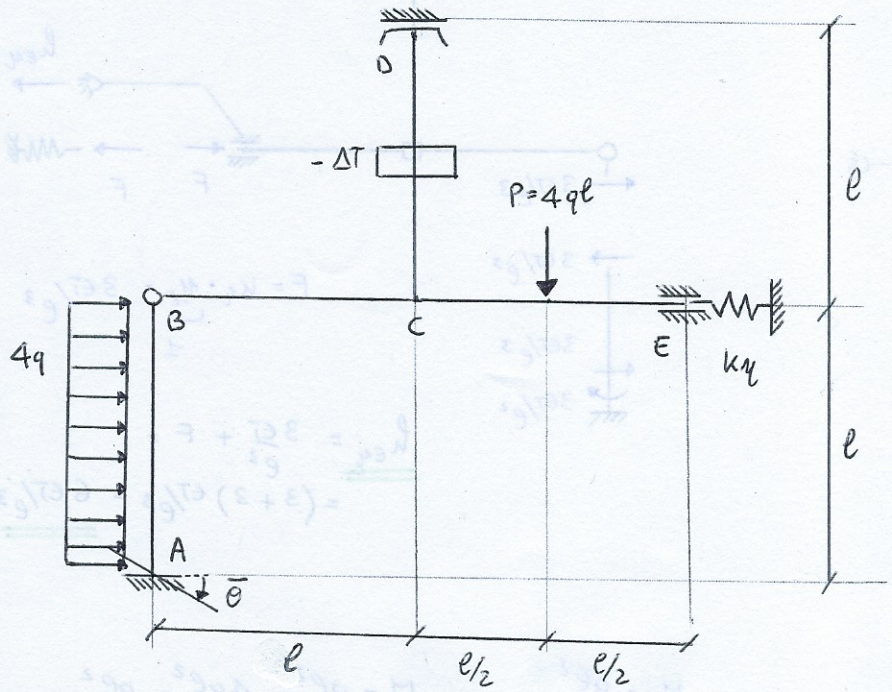
Dato il telaio in figura, **si richiedono i grafici di:**

1. Momento flettente (con il valore e la posizione dei massimi);
2. Taglio;
3. Azione assiale;
4. Deformata qualitativa con posizione dei flessi.

Si assuma $EA \to \infty$, $EJ = costante$.

I grafici possono essere realizzati in matita, mentre i calcoli necessari per lo sviluppo del tema devono essere in tratto non cancellabile. Il tutto deve essere riportato chiaramente.

TEMA ESAME - TELAIO 22/11/2021



DATI:

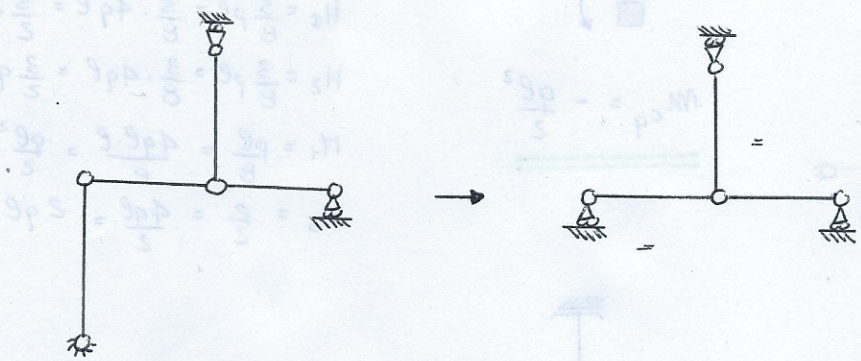
- $\angle\Delta T = \frac{1}{2}\frac{q\ell^3}{EJ}$
- $k_\varphi = 3\,EJ/\ell^3$
- $\bar{\theta} = \frac{q\ell^3}{EJ}$

GRADI DI LIBERTA' → $3+3=6$

GRADI DI VINCOLO → $3+2+2+2+1=10$

STRUTTURA 4 VOLTE IPERSTATICA

LABILITA'



SPOSTAMENTO ORIZZONTALE CONSENTITO, STRUTTURA A NODI SPOSTABILI

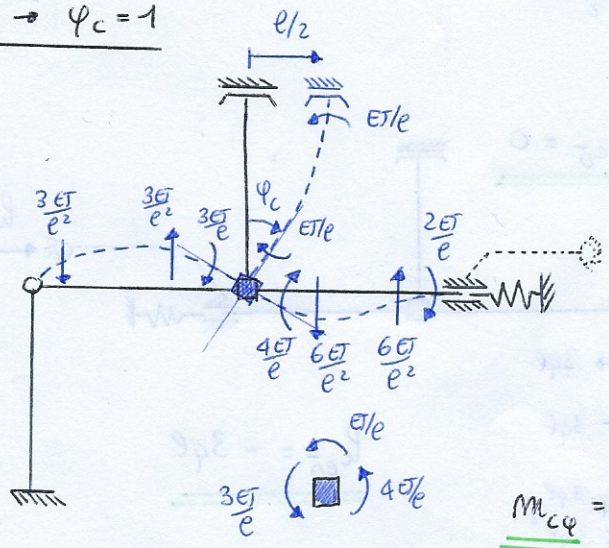
INCOGNITE: φ_C ; M_E

CONVENZIONI

SISTEMA RISOLUENTE

$$\begin{cases} m_{C\varphi}\cdot\varphi_C + m_{CM}\cdot M_E + m_{C\phi} = 0 \\ h_{E\varphi}\cdot\varphi_C + h_{EM}\cdot M_E + h_{E\phi} = 0 \end{cases}$$

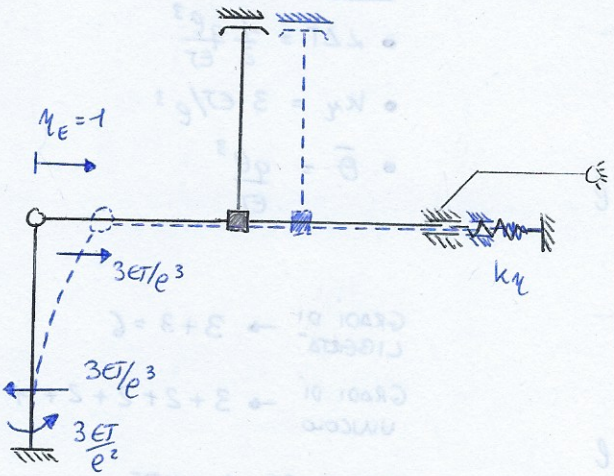
CASO 1 → $\varphi_C = 1$



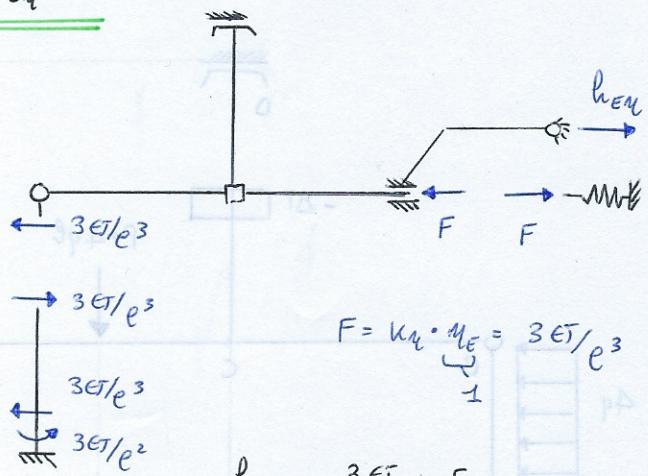
$h_{E\varphi} = 0$

$m_{C\varphi} = (1+3+4)\,EJ/\ell = 8\,EJ/\ell$

CASO 2 → $\eta_E = 1$



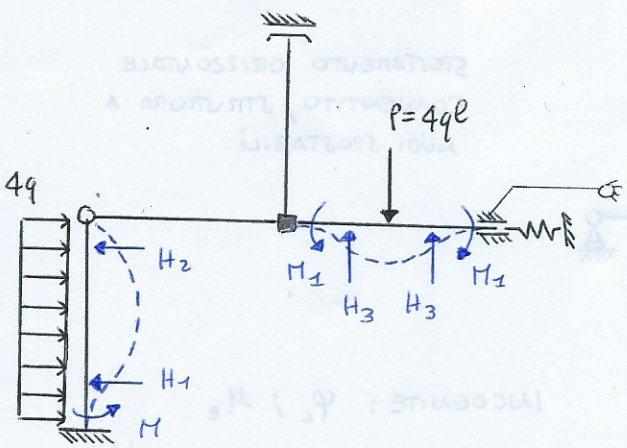
$m_{c_M} = 0$



$F = k_M \cdot \eta_E = 3EJ/\ell^3$ ($\eta_E = 1$)

$h_{EM} = \frac{3EJ}{\ell^3} + F = (3+3)\,EJ/\ell^3 = 6EJ/\ell^3$

CASO 3 → $q \neq 0$



$M_1 = \frac{q\ell^2}{2}$

$m_{c_q} = -\frac{q\ell^2}{2}$

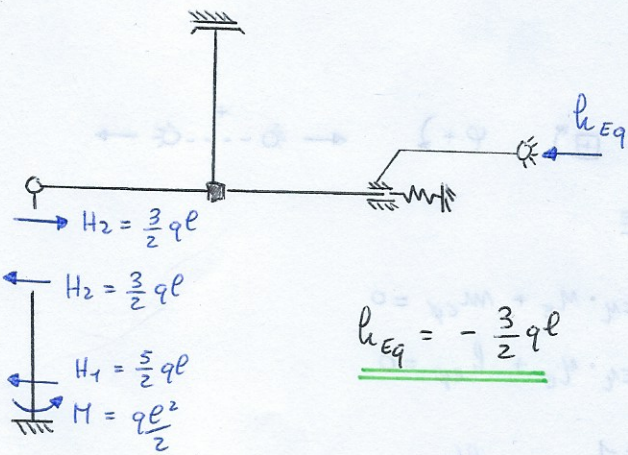
$M = \frac{p\ell^2}{8} = \frac{4q\ell^2}{8} = \frac{q\ell^2}{2}$

$H_1 = \frac{5}{8}p\ell = \frac{5}{8}\cdot 4q\ell = \frac{5}{2}q\ell$

$H_2 = \frac{3}{8}p\ell = \frac{3}{8}\cdot 4q\ell = \frac{3}{2}q\ell$

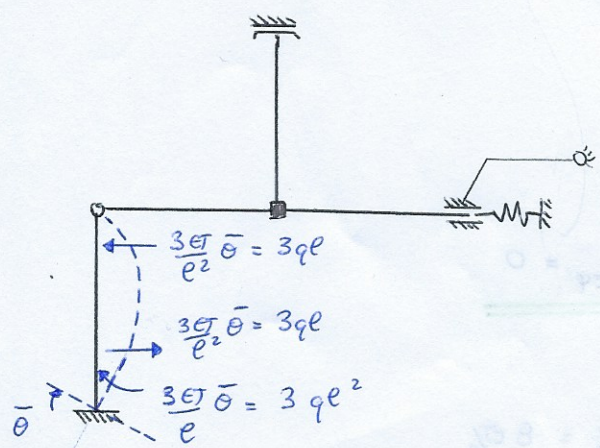
$M_1 = \frac{p\ell}{8} = \frac{4q\ell\cdot\ell}{8} = \frac{q\ell^2}{2}$

$H_3 = \frac{p}{2} = \frac{4q\ell}{2} = 2q\ell$

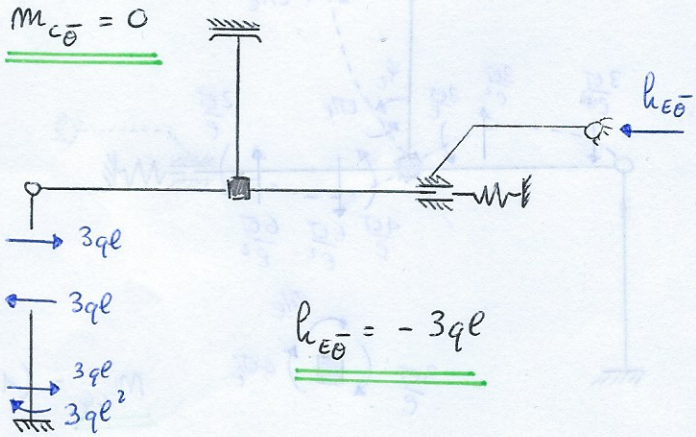


$h_{Eq} = -\frac{3}{2}q\ell$

CASO 4 → $\bar{\theta} \neq 0$



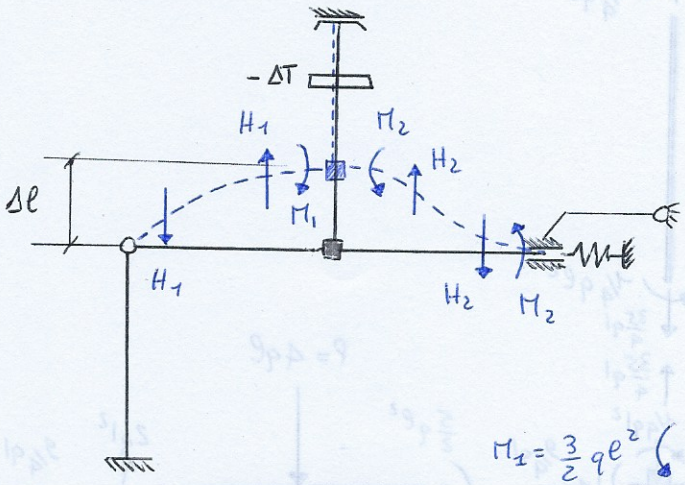
$m_{c_{\bar{\theta}}} = 0$



$h_{E\bar{\theta}} = -3q\ell$

CASO 5 → $\Delta T \neq 0$

$$\Delta \ell = \lambda \Delta T \ell = \frac{1}{2}\frac{q\ell^3}{EJ}\cdot \ell = \frac{1}{2}\frac{q\ell^4}{EJ}$$



$$H_1 = \frac{3EJ}{\ell^3}\cdot\Delta\ell = \frac{3EJ}{\ell^3}\cdot\frac{1}{2}\frac{q\ell^4}{EJ} = \frac{3}{2}q\ell$$

$$M_1 = \frac{3EJ}{\ell^2}\cdot\Delta\ell = \frac{3EJ}{\ell^2}\cdot\frac{1}{2}\frac{q\ell^4}{EJ} = \frac{3}{2}q\ell^2$$

$$H_2 = \frac{12EJ}{\ell^3}\cdot\Delta\ell = \frac{12EJ}{\ell^3}\cdot\frac{1}{2}\frac{q\ell^4}{EJ} = 6q\ell$$

$$M_2 = \frac{6EJ}{\ell^2}\cdot\Delta\ell = \frac{6EJ}{\ell^2}\cdot\frac{1}{2}\frac{q\ell^4}{EJ} = 3q\ell^2$$

$M_1 = \frac{3}{2}q\ell^2$ (■) $M_2 = 3q\ell^2$

$$m_{c_{\Delta T}} = +\frac{3}{2}q\ell^2 - 3q\ell^2 = -\frac{3}{2}q\ell^2$$

$$h_{\epsilon_{\Delta T}} = 0$$

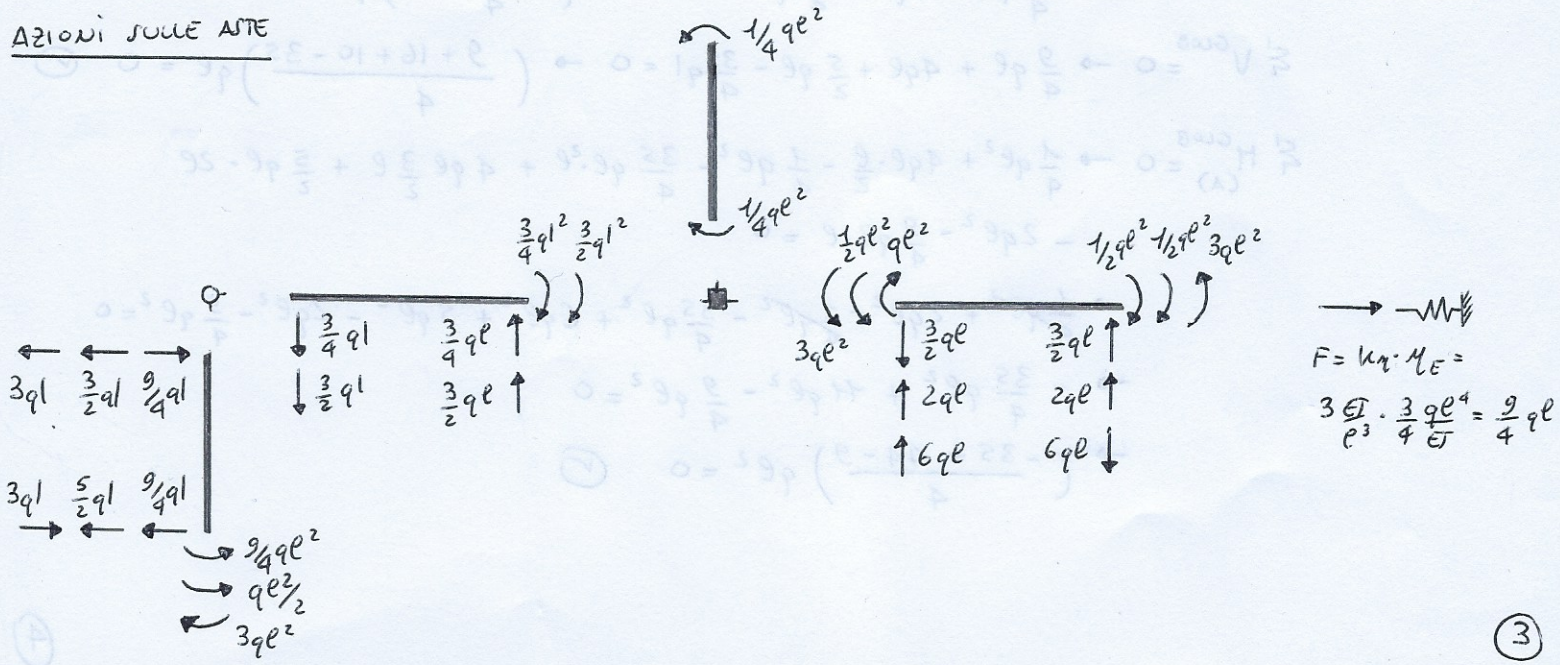
SISTEMA RISOLUENTE

$$\begin{cases} m_{c\varphi}\cdot\varphi_c + m_{cM}\cdot M_\epsilon + m_{c\phi} = 0 \\ h_{\epsilon\varphi}\cdot\varphi_c + h_{\epsilon M}\cdot M_\epsilon + h_{\epsilon\phi} = 0 \end{cases}$$

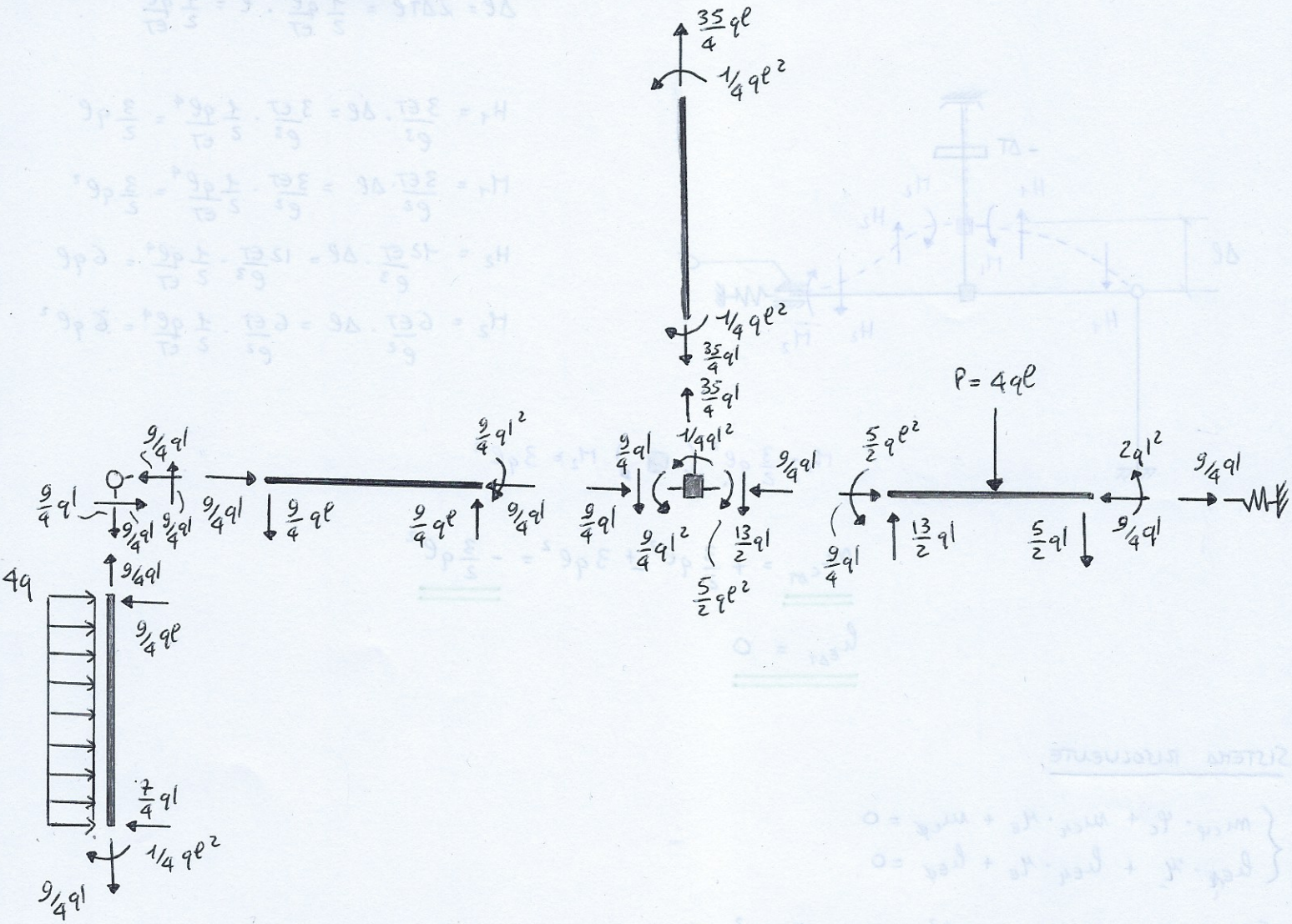
$$\begin{cases} 8\frac{EJ}{\ell}\cdot\varphi_c + 0\cdot M_\epsilon - \frac{q\ell^2}{2} + 0 - \frac{3}{2}q\ell^2 = 0 \\ 0\cdot\varphi_c + 6\frac{EJ}{\ell^3}\cdot M_\epsilon - \frac{3}{2}q\ell - 3q\ell = 0 \end{cases}$$

$$\begin{cases} 8\frac{EJ}{\ell}\cdot\varphi_c - 2q\ell^2 = 0 \rightarrow \boxed{\varphi_c = \frac{1}{4}\frac{q\ell^3}{EJ}} \\ 6\frac{EJ}{\ell^3}\cdot M_\epsilon - \frac{9}{2}q\ell = 0 \rightarrow M_\epsilon = \frac{9}{2}q\ell\cdot\frac{\ell^3}{6EJ} \rightarrow \boxed{M_\epsilon = \frac{3}{4}\frac{q\ell^4}{EJ}} \end{cases}$$

AZIONI SULLE ASTE



AZIONI COMPLESSIVE SULLE ASTE



EQUILIBRI

- AL NODO → $\Sigma' M_{(C)} = \left(\frac{9}{4} + \frac{1}{4} - \frac{5}{2}\right) ql^2 = 0$ ✓

  $\Sigma' H_{(C)} = \left(\frac{9}{4} - \frac{9}{4}\right) ql = 0$ ✓

  $\Sigma' V_{(C)} = \left(\frac{35}{4} - \frac{13}{2} - \frac{9}{4}\right) ql = 0$ ✓

- GLOBALE → $\Sigma' H^{GLOB} = 0 \rightarrow \frac{7}{4} ql - 4ql + \frac{9}{4} ql = 0 \rightarrow \left(\frac{7 - 16 + 9}{4}\right) ql = 0$ ✓

  $\Sigma' V^{GLOB} = 0 \rightarrow \frac{9}{4} ql + 4ql + \frac{5}{2} ql - \frac{35}{4} ql = 0 \rightarrow \left(\frac{9 + 16 + 10 - 35}{4}\right) ql = 0$ ✓

  $\Sigma' M^{GLOB}_{(A)} = 0 \rightarrow \frac{1}{4} ql^2 + 4ql \cdot \frac{l}{2} - \frac{1}{4} ql^2 - \frac{35}{4} ql \cdot l + 4ql \frac{3}{2} l + \frac{5}{2} ql \cdot 2l$

  $- 2ql^2 - \frac{9}{4} ql \cdot l = 0$

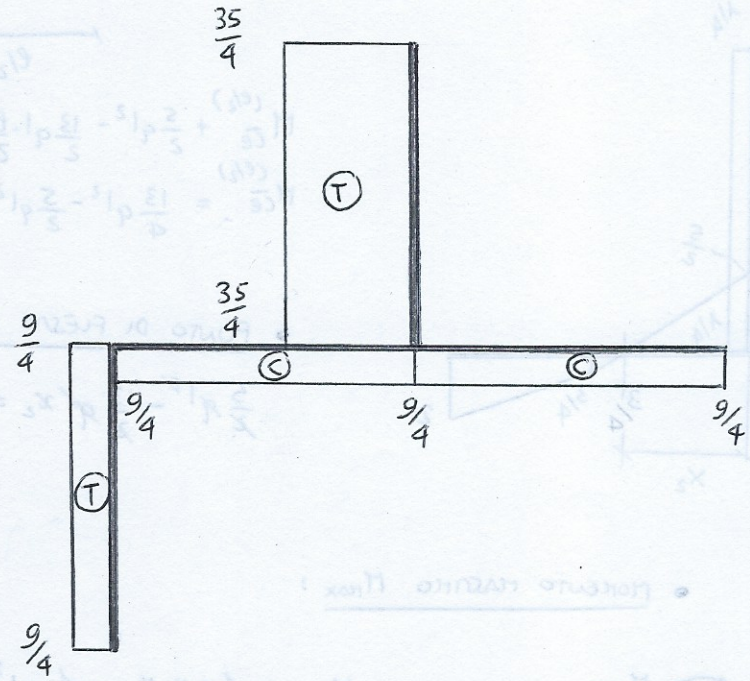
  $\rightarrow \cancel{\frac{1}{4} ql^2} + 2ql^2 - \cancel{\frac{1}{4} ql^2} - \frac{35}{4} ql^2 + 6ql^2 + 5ql^2 - 2ql^2 - \frac{9}{4} ql^2 = 0$

  $\rightarrow - \frac{35}{4} ql^2 + 11 ql^2 - \frac{9}{4} ql^2 = 0$

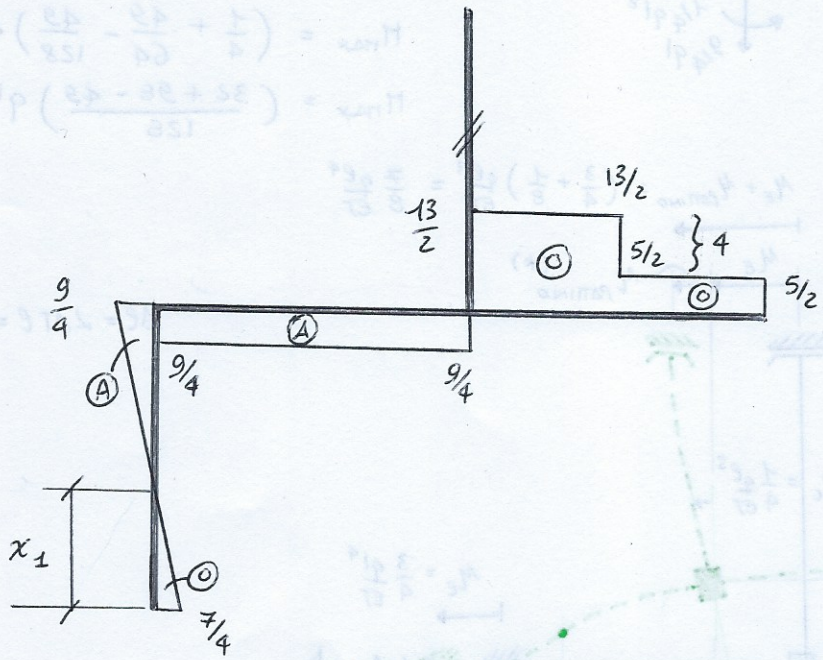
  $\rightarrow \left(\frac{-35 + 44 - 9}{4}\right) ql^2 = 0$ ✓

DIAGRAMMI

$\left[\frac{N}{qe}\right]$



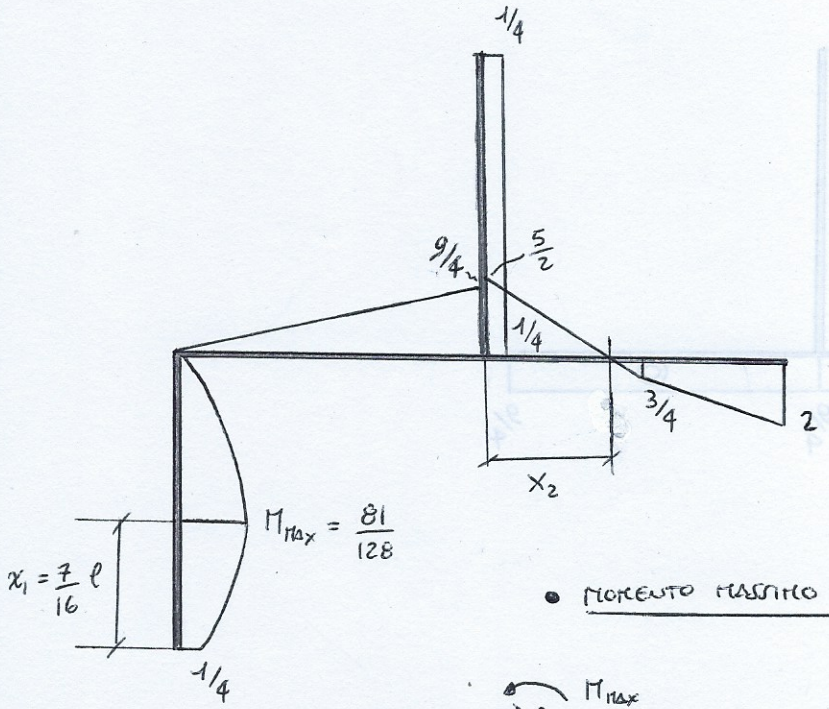
$\left[\frac{T}{qe}\right]$



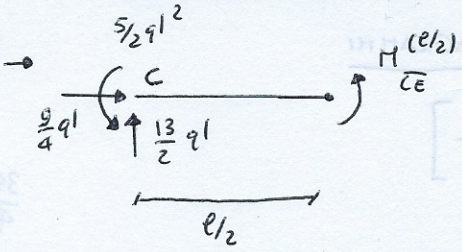
- ANNULLAMENTO DEL TAGLIO → M_{MAX}

$$\frac{7}{4}ql - 4q x_1 = 0 \rightarrow x_1 = \frac{7}{4}e \cdot \frac{1}{4} \rightarrow x_1 = \frac{7}{16}e$$

$\left[\frac{M}{ql^2}\right]$



- $M_{\overline{CE}}^{(\ell/2)}$ →



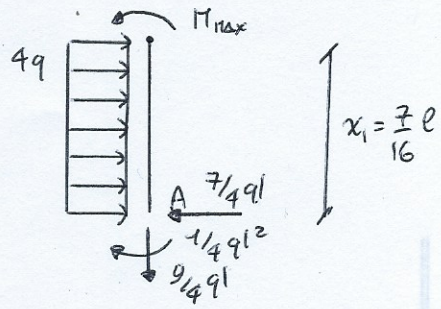
$$M_{\overline{CE}}^{(\ell/2)} + \frac{5}{2}ql^2 - \frac{13}{2}ql\cdot\frac{\ell}{2} = 0$$

$$M_{\overline{CE}}^{(\ell/2)} = \frac{13}{4}ql^2 - \frac{5}{2}ql^2 \rightarrow M_{\overline{CE}}^{(\ell/2)} = \frac{3}{4}ql^2$$

- PUNTO DI FLESSO x_2 :

$$\frac{5}{2}ql^2 - \frac{13}{2}qx_2 = 0 \rightarrow x_2 = \frac{5}{13}\ell$$

- MOMENTO MASSIMO M_{max} :



$$M_{max} + 4q\,x_1\frac{x_1}{2} - \frac{1}{4}ql^2 - \frac{7}{4}q\ell\,x_1 = 0 \rightarrow$$

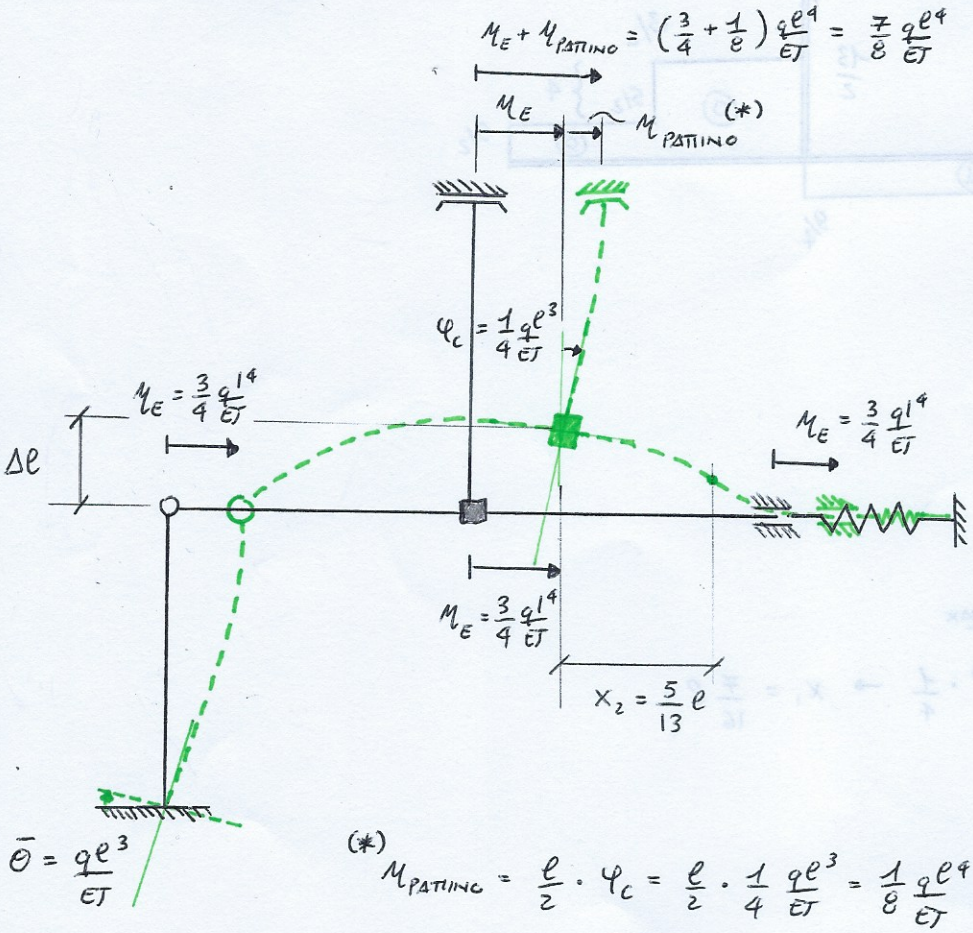
$$M_{max} + 2qx_1^2 - \frac{1}{4}ql^2 - \frac{7}{4}q\ell x_1 = 0 \rightarrow$$

$$M_{max} + \frac{49}{128}ql^2 - \frac{1}{4}ql^2 - \frac{49}{64}ql^2 = 0 \rightarrow$$

$$M_{max} = \left(\frac{1}{4} + \frac{49}{64} - \frac{49}{128}\right)ql^2 \rightarrow$$

$$M_{max} = \left(\frac{32 + 98 - 49}{128}\right)ql^2 \rightarrow M_{max} = \frac{81}{128}ql^2$$

DEFORMATA QUALITATIVA



$$\Delta\ell = 2\Delta T\,\ell = \frac{1}{2}\frac{q\ell^4}{EJ}$$

(*) $$M_{PATTINO} = \frac{\ell}{2}\cdot\varphi_C = \frac{\ell}{2}\cdot\frac{1}{4}\frac{q\ell^3}{EJ} = \frac{1}{8}\frac{q\ell^4}{EJ}$$